Assignment sheet 1

September 20, 2017

Assignment 1. A, B and C are events such that

$$\begin{aligned} P(A) &= 0.4, & P(B \cap C) &= 0.2, \\ P(B) &= 0.7, & P(C \cap (A \cup B)) &= 0.2, \\ P(C) &= 0.3, & P(B \cap (A \cup C)) &= 0.4, \quad \text{and} \\ P(A \cup B) &= 0.8. \end{aligned}$$

(a) Find the probability that exactly two of A, B and C occur.
(b) Find the probability that none of A, B and C occur.
(c) Find the probability that A and exactly one of B and C occur.

Assignment 2. Three students, A, B, C, have equal claims for an award. They decide that each will toss a coin, and that the man whose coin falls unlike the other two wins. (The 'odd man' wins.) If all three coins fall alike, they toss again.
(a) Describe a sample space for the result of the first toss of the three coins.
(b) Assign probabilities to the elements of the sample space.
(c) What is the probability that A wins on the first toss ? That B does ? That C does ?
(d) What is the probability that there is no winner on the first toss ?
(e) Given that there is a winner on the first toss, what is the probability that it is A ?

Assignment 3. Three prisoners in solitary confinement, A, B and C, have been sentenced to death on the same day but, because there is a national holiday, the governor decides that one will be granted a pardon. The prisoners are informed of this but told that they will not know which one of them is to be spared until the day scheduled for the executions.
Prisoner A says to the jailer "I already know that at least one the other two prisoners will be executed, so if you tell me the name of one who will be executed, you won't have given me any information about my own execution".
The jailer accepts this and tells him that C will definitely die.
A then reasons "Before I knew C was to be executed, I had a 1 in 3 chance of receiving a pardon. Now I know that either B or myself will be pardoned. So, the odds have improved to 1 in 2".
But the jailer points out "You could have reached a similar conclusion if I had said B will die, and I was bound to answer either B or C, so why did you need to ask ?".
(a) What are A's chances of receiving a pardon and why ?
(b) What are B's chances of receiving a pardon and why ?
(c) Suppose now that there are n prisoners $A_1, A_2, \ldots, A_n$. Prisoner A_1 asks the jailer to tell the names of $n-2$ other prisoners (except him) who will be executed. The jailer accepts this and tells him that $A_3, A_4, \ldots, A_n$ will definitely be executed. What are A_1's chances of receiving a pardon and why ? What about A_2 ?

Assignment 4. A blood test for a particular medical condition turn out either 'positive' or 'negative'. 'Positive' indicates that the person tested has the disease in question, while 'negative' indicates that he does not have it. Suppose that such a test for this disease sometimes makes mistakes : 1 in 100 of those free of the disease have 'positive' test results, and 2 in 100 of those having the disease have 'negative' test results. The rest are correctly identified. The disease is also quite rare : one person in 1000 has the disease. Find the probability that a person with a 'positive' test result actually has the disease. Also, find the probability that a person with a 'negative' test result does not actually have the disease. Comment on your findings.

Assignment 5. (a) Use the command

```
set.seed(20092017)
runif(n,0,1)
```

to generate $n = 1000$ i.i.d. samples from a Unif(0,1) distribution. Store the values as a vector X.

(b) Let $q_\lambda(x) = -log_e(1-x)/\lambda, x \in (0,1)$. Write a user-defined function called "quant" that will compute the value of $q_\lambda(x)$ for a given $x \in (0,1)$ and $\lambda \in (0,\infty)$.

(c) For each $\lambda \in \{1,2,4\}$, compute the vectors $q_\lambda(X)$ as $Y\lambda$.

(d) For each $\lambda \in \{1,2,4\}$, compute the empirical cdf of the sample $Y\lambda$ using the command `ecdf(Yλ)` and store it as $E\lambda$.

(e) Run the following command :

```
par(mfrow=c(1,3))
plot(E1)
plot(E2)
plot(E3)
```

Interpret what you see.

(f) Let

$$F_\lambda(x) = \begin{cases} 1 - exp(-\lambda x), & x \in [0,\infty) \\ 0, & x < 0 \end{cases}$$

denote the cdf of the Exp(λ) distribution. For each $\lambda \in \{1,2,4\}$, write a user-defined function called "cdfλ" that will compute the value of $F_\lambda(x)$ for a given $x \in (0,1)$.

(g) Run the following command :

```
par(mfrow=c(1,3))
curve(cdf1,-1,5)
curve(cdf2,-1,5)
curve(cdf3,-1,5)
```

Interpret what you see. Save the output as a pdf with file name "plot1.pdf".

(h) Run the following command :

```
par(mfrow=c(1,3))
curve(E1,-1,5)
curve(cdf1,-1,5,add=TRUE,col="red")
curve(E2,-1,5)
curve(cdf2,-1,5,add=TRUE,col="red")
curve(E3,-1,5)
curve(cdf3,-1,5,add=TRUE,col="red")
```

Save the output as a pdf with file name "plot2.pdf".

(i) Interpret what you see. What does it tell you about the sampling distribution of $Y\lambda$? *(The reason for what you see will be clear in due course of time).*

(j) What is the relation between q_λ and F_λ?

(k) Will the observations made in above still hold if F_λ is replaced by any other continuous, strictly increasing cdf?

(l) Verify your assertion in (k) by replacing F_λ and q_λ with the in-built functions `pnorm` and

`qnorm`, respectively. These are the cdf and the quantile function, respectively, of the N(0,1) distribution.
(Note : unlike F_λ and q_λ, the functions `pnorm` and `qnorm` have no closed form expressions).